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INTERFACIAL LEVEL GRADIENT EFFECTS IN HORIZONTAL NEWTONIAN LIQUID–GAS STRATIFIED FLOW—I

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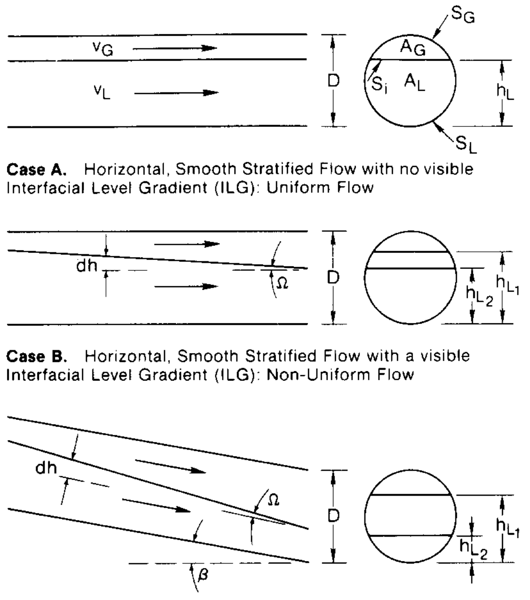
Abstract—The analysis of reported Newtonian liquid–gas stratified flow data for horizontal circular ducts indicated that an interfacial level gradient (ILG) and therefore non-uniform flow tended to exist over a wide range of test conditions. Significant ILG can be present if high-viscosity liquids and low gas velocities are used to produce stratified flow. ILG can reduce the liquid holdup and can possibly expand the stratified flow regime by delaying the transition to wavy stratified and/or intermittent flow. Use of the Lockhart–Martinelli parameters ϕ_L^2 and ϕ_G^2 is invalid in stratified flow if ILG is present because of unequal axial pressure gradients in the gas and liquid phases. During uniform stratified flow, especially in the laminar liquid–turbulent gas flow regime, the combined one-dimensional mechanical energy equations can be used in dimensionless form to accurately predict the liquid holdup and pressure drop. In future stratified flow experiments, the axial pressure gradient in both phases should be measured.

INTRODUCTION

Most reported liquid–gas stratified flow experimental results have been concerned primarily with uniform flow and have not given much attention to the influence of a change in the liquid depth along the channel. Flow with an interfacial level gradient (ILG), which is a special type of non-uniform stratified flow, can possibly affect liquid holdup, flow-pattern transition and the assumption of an equal axial pressure gradient in each phase. Moreover, as in open-channel flow, the sign of the ILG should be determined by whether the flow is subcritical or supercritical.

Figure 1 illustrates three cases of smooth stratified flow. Case A represents nearly uniform horizontal flow where the ILG is not steep enough to be observed. Case B depicts horizontal stratified flow having an ILG which is observable. Case C includes both ILG and tube inclination. Both Case A and Case B are far removed from entrance or exit influence where rapidly varying flow does not exist (such as flow at a free overflow). Gradually varied flow is assumed in Case B. Either Case A, B or C can be wavy stratified (WS) as well as smooth stratified (SS) flow. Past experience has demonstrated that gradually varied non-uniform flow can be analyzed using the one-dimensional energy equation for open-channel steady flow through circular ducts (Chow 1959; Henderson 1966).

One objective of this work is to analyze the reported Newtonian liquid–gas stratified flow data for horizontal circular ducts to determine the effect that ILG has on liquid holdup R_L, flow-pattern transition, the two-phase pressure drop parameter ϕ_L^2 (ratio of the two-phase to single-phase liquid pressure gradient) and critical channel flow behavior. Another objective is to check the general validity of the one-dimensional energy equations for a wide range of liquid–gas stratified flow data. These analyses focused on previous liquid–gas experimental results employing high-viscosity liquids in laminar flow through large-diameter circular tubes; however, such data were extremely limited. The interest was initiated by the background needs of non-Newtonian liquid–gas stratified flow tests, conducted by the authors and reported in the accompanying paper (Bishop & Deshpande 1986, this issue, pp. 977–996).



Case A. Horizontal, Smooth Stratified Flow with no visible Interfacial Level Gradient (ILG): Uniform Flow

Case B. Horizontal, Smooth Stratified Flow with a visible Interfacial Level Gradient (ILG): Non-Uniform Flow

Case C. Inclined, Smooth Stratified Flow with a visible Interfacial Level Gradient: Non-Uniform Flow

Figure 1. Uniform and non-uniform smooth stratified flow.

PREVIOUS WORK

Although the experimental data analyzed covered a 35-year period, the experiments which reported both liquid holdup and pressure drop and which used high-viscosity liquids and tubes larger than 0.0254 m are surprisingly limited, as illustrated in summary tables 1 and 2. Stratified flow is conceptually simple, however, the low liquid flow rates involved require accurate low-range flow-measuring instrumentation. Moreover, stratified flow stability is often more sensitive to vibrational effects compared to other types of two-phase flow patterns. Finally ILG and critical flow considerations add to the analysis complexities.

The type of device used to bring the two phases together can influence downstream behavior. Either some form of a mixing tee (MT) or a combining tee (CT) is used. The

Table 1. Stratified flow experimental geometric parameters

	TS length (m)	ILG reported	Entrance length (m)	Exit length (m)	Pressure drop measured	Type of combining tee used	ILG measured	Length / Diameter
Holden (1948)	4.6	Yes	0.91	0.6	Gas	SCT	Yes	245
Bergelin & Gazley (1949)	4.6	Yes	0.91	0.6	Gas	SCT	Yes	245
Hoogendoorn (1959)	8.0	No	10.0	7.0	Uncertain	CT	No	190
Govier & Omer (1962)	9.1	No	2.42	0.9	Gas	CT	No	478
Jensen (1972)	7.3	No	0.9	0.9	Gas	CT	Yes	358 239 178
Agrawal *et al.* (1973)	17.0	No	6.9	6.7	CC	CT	No	1170
Arruda (1970)	7.3	No	0.9	0.9	Gas	CT	Yes	360 180
Weisman *et al.* (1979)	6.0	No	—	—	None	CT	No	—
Simpson *et al.* (1976, 1981)	16.0	Yes	—	—	Gas CC Liquid	MT	No	—

Table 2. Stratified flow experimental fluid parameters

	Tube diameter (m)	Type of liquid used	Type of gas used	Liquid viscosity range (cP)	Liquid velocity (V_{SL}) range (m/s)	Gas velocity (V_{SG}) range (m/s)	Flow pattern observed	Flow pattern predicted
Holden (1948)	0.026	Water	Air	1.0	0.031	0.61	S-ILG	—
							SS	
					0.085	3.36	WS	
Bergelin & Gazley	0.051	Water	Air	1.0	0.014	0.36	S-ILG	I
(1949)							SS	SS
					0.61	3.25	WS	WS
Hoogendoorn	0.14	Oil	Air	20	0.072	3.2–21.0	S-ILG	WS
(1959)							WS	SS
Govier & Omer	0.0254	Water	Air	1.0	0.0092	0.0157–3.02	SS	SS
(1962)							WS	WS
Jensen (1972)	0.0254	W		0.8–310	0.00075	2.0	S-ILG	
	0.0381		Air					SS
	0.0508	G–W			0.1	9.1	SS	
Agrawal *et al.* (1973)	0.026	Oil	Air	5.1	0.014	0.11	SS	
								SS
					0.061	6.1	WS	
Arruda (1970)	0.0254	G–W	Air	3.5	0.0091–0.057	1.21–4.14	SS–SW	SS
							transition	
	0.0508							
Weisman *et al.*	0.0508	G–W	Air	75 and 150	0.07–0.14	0.17–4.8	SS	I
(1979)								
Simpson *et al.*	0.127	Water	Air	1.0	0.094–0.52	0.012–1.47	SS	I
(1976, 1981)	0.216						SW	SS
								WS

term combining tee is used here to define situations where the two phases are not physically mixed; in contrast, a mixing tee physically mixes the two phases. If high-viscosity liquids are used, it is often not practical to employ an MT because of the longer tube length required to disengage the fluids. As seen in tables 1 and 2, most experimenters used simple CTs. Only Simpson *et al.* (1976, 1981) used an MT. Holden (1948) and Gazley (1948, 1949) used a special combining tee (SCT) which appeared to promote early stabilization of stratified flow.

Holden (1948), Bergelin & Gazley (1949) and Gazley (1948, 1949) were the first workers to call attention to the unequal two-phase axial pressure gradients which can exist in stratified flow as a result of non-uniform flow ILG behavior. Holden and Gazley used the same equipment and measured the liquid level along the test section and the axial static pressure only in the gas phase. Jensen (1972) and Arruda (1970) employed both mechanical and electronic techniques to measure the liquid level but surprisingly they did not discuss the non-uniform flow behavior. They measured the axial pressure gradient in the gas phase. Govier & Omer (1962) measured only the pressure gradient in the gas phase. Other than Holden and Gazley none of the early experimenters reported measurements of ILG or made reference to the presence of ILG. Recently, Simpson *et al.* (1976, 1981) measured the axial pressure gradient in the gas phase, in the liquid phase and along the centerline in large-diameter tubes. Simpson called special attention to ILG and to the large differences in the two pressure drop measurements and the error introduced by using centerline pressure taps if significant ILG exists. Although Weisman *et al.* (1979) reported only flow-pattern information, they used high-viscosity liquids and their results illustrate the effect that ILG can have on flow-pattern transition. Agrawal (1971; Agrawal *et al.* 1973) used centerline taps to measure the axial pressure gradient. No experiments were found in which the gas-phase pressure gradient, liquid-phase pressure gradient and ILG were measured simultaneously.

ANALYSIS

The holdup and pressure drop data reported were used to determine whether ILG existed, its relative magnitude and its influence on the principal hydrodynamic parameters. If ILG profiles were reported, assessment of subcritical and supercritical flow was made and compared to open-channel flow predictions. Comparison of the interfacial shear stresses τ_{iL} and τ_{iG} was also made.

Referring to figure 1, the one-dimensional energy equations can be written as follows.

Case A. Horizontal, stratified, uniform flow; no ILG assumed

$$-\left(\frac{dP}{dx}\right)_{TPL} = \frac{\tau_{WL}S_L}{A_L} - \frac{\tau_{iL}S_i}{A_L} \qquad [1]$$

and

$$-\left(\frac{dP}{dx}\right)_{TPG} = \frac{\tau_{WG}S_G}{A_G} + \frac{\tau_{iG}S_i}{A_G}, \qquad [2]$$

where

$$V_{SG} \gg V_{SL} \quad \text{and} \quad \left(\frac{dP}{dx}\right)_{TPL} = \left(\frac{dP}{dx}\right)_{TPG}.$$

Here, dP/dx is the axial pressure gradient, τ is the shear stress, S is the perimeter, A is the cross-sectional area for flow and V is the fluid velocity. The subscript TP stands for two-phase, W for wall, i for interfacial conditions and S for superficial quantity. Moreover, subscripts L and G are used to denote the liquid and gas phase, respectively, throughout the paper.

If the axial two-phase pressure gradient is known in one phase and if the holdup is also known, [1] and [2] can be solved, assuming equal interfacial shear stresses to determine if the axial pressure gradients in the two phases are equal. A significant difference between $(dP/dx)_{TPL}$ and $(dP/dx)_{TPG}$ indicates the presence of ILG. Alternatively, if the two-phase pressure gradients are assumed to be equal, a large difference between τ_{iG} and τ_{iL} suggests ILG is present. When calculating the *in situ* laminar flow τ_{WG} or τ_{WL}, a shape factor is not required because Straub *et al.* (1958) reported that for laminar flow through partially-filled smooth circular ducts, the friction factors group around 16/Re, where Re is the Reynolds number.

Taitel & Dukler (1976a) combined [1] and [2] to produce the dimensionless equation

$$\chi^2 F(R_L, n, m) - F\left(R_L, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right) = 0, \qquad [3]$$

where F is a functional relationship involving the parameters within the brackets; n and m (exponents of the Reynolds number in the friction factor relationship) simply depend on whether the liquid and gas phases are laminar or turbulent, R_L is the liquid holdup, f is the friction factor and χ is the Lockhart–Martinelli parameter. If values for the interfacial friction factors f_{iG} and f_{iL} are assumed, [3] can be iterated to obtain holdup R_L. Because [3] is valid only for smooth or wavy uniform stratified flow, if $f_{iL} = f_{iG}$ is assumed, deviations of experimental holdup values from those predicted by [3] indicate wavy stratified flow or stratified flow with ILG. Equation [3] is used generally in subsequent analyses as a basis for determining the presence of ILG.

Case B. Horizontal stratified flow with ILG

Again referring to figure 1, the one-dimensional energy equations are

$$-\left(\frac{dP}{dx}\right)_{TPL} - g\rho_L \frac{dh_L}{dx} - \frac{\alpha\rho_L}{2}\frac{d(V_L^2)}{dx} = \frac{\tau_{WL}S_L}{A_L} - \frac{\tau_{iL}S_i}{A_L} \qquad [4]$$

and

$$-\left(\frac{dP}{dx}\right)_{TPG} - \frac{\alpha\rho_G}{2}\frac{d(V_G^2)}{dx} = \frac{\tau_{WG}S_G}{A_G} + \frac{\tau_{iG}S_i}{A_G}, \qquad [5]$$

where, g is the acceleration due to gravity, ρ is the density and h_L is the height of liquid in the tube.

The parameter α accounts for radial variation in the velocity profile and is 2 for laminar flow of a Newtonian fluid through a circular tube. Only limited information exists to define α in laminar open-channel flow. If the pressure gradient is measured in each phase, the ratio $[(\Delta P/\Delta L)_{TPLM}/(\Delta P/\Delta L)_{TPGM}]$ is a quantitative measurement of (dh_L/dx), the ILG ($\Delta P/\Delta L$ is the pressure gradient and the subscripts TPLM and TPGM indicate the measured two-phase quantity for the liquid and gas phase, respectively). In general, the liquid kinetic energy term in [4] is very small and the gas kinetic energy term is $< 10\%$ of the l.h.s. of [5]. Thus, where there is ILG and only the pressure gradients are measured, [4] and [5] can be used to determine the influence of ILG on the ratio of τ_{iL}/τ_{iG} because the interfacial shear stress terms can be calculated directly if the holdup is known. Measurement of only $(dP/dx)_{TPG}$, as is often the case, still allows a quantitative assessment of ILG to be made using [4] and [5] if it is assumed that $\tau_{iL} = \tau_{iG}$.

Equations [4] and [5] can be put in a dimensionless form similar to [3]. Let

$$I = -\left(\frac{dP}{dx}\right)_{TPL} - g\rho_L\left(\frac{dh_L}{dx}\right) - \frac{\alpha\rho_L}{2}\frac{d(V_L^2)}{dx} + \left(\frac{dP}{dx}\right)_{TPG} + \frac{\alpha\rho_G}{2}\frac{d(V_G^2)}{dx}, \qquad [6]$$

by combining [4] and [5], [7] is obtained:

$$I + \frac{\tau_{WG}S_G}{A_G} - \frac{\tau_{WL}S_L}{A_L} + \frac{\tau_{iL}S_i}{A_L} + \frac{\tau_{iG}S_i}{A_G} = 0 \qquad [7]$$

which in dimensionless form gives

$$\chi^2 F(R_L, n, m) - F\left(R_L, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right) - Z = 0, \qquad [8a]$$

where

$$\chi^2 = \frac{-\left(\dfrac{dP}{dx}\right)_{SL}}{-\left(\dfrac{dP}{dx}\right)_{SG}} \quad \text{(Lockhart–Martinelli parameter)}$$

and

$$Z = \frac{4I}{F(R_L)\left(\dfrac{dP}{dx}\right)_{SG}}. \qquad [8b]$$

The parameter Z represents the equivalent relative dimensionless force acting on the liquid in the direction of flow due to the ILG and any other difference in the two-phase pressure gradient in each phase. Taitel & Dukler (1976a) obtained an equation similar to [8a] for the case of uniform inclined stratified flow and introduced a parameter Y to account for tube inclination. Both Z and Y can have a similar effect on the hydraulics of stratified flow. However, whereas Y is uniquely determined from the inclination angle, Z is more complicated and cannot be assigned a unique value *a priori*. Analysis suggests that Z, which is related to ILG, may be dependent on the system hydrodynamic parameters including entrance conditions, exit conditions, tube length and tube diameter. Barnea *et al.* (1980) have shown that for downward inclined flow, only a slight inclination, less than 1°, expands the stratified flow regime by a factor of 3–5 for air–water flowing through a 0.0254 m dia tube. Therefore, it might also be expected that a downward ILG expands the stratified flow regime. Equation [8a] shows that for fixed values of χ, n, m and the interfacial friction factors, the effect of Z is to lower the holdup from that predicted for uniform stratified flow. Because wave formation would tend to be delayed for a shallower liquid depth, the transition to wavy stratified flow and subsequently to intermittent flow could be delayed.

If holdup data and the interfacial shear stresses are known or assumed, [8a] can be iterated to obtain Z because in that case there would only be one unknown. The magnitude of Z is a measure of the deviation from uniform flow. The parameter Z, or the equivalent I, would then be correlated using χ and/or R_L.

Case C. Inclined stratified flow with ILG

The one-dimensional energy equations are

$$-\left(\frac{dP}{dx}\right)_{TPL} - \rho_L g \frac{dh_L}{dx} - \frac{\alpha\rho_L}{2}\frac{d(V_L^2)}{dx} = \frac{\tau_{WL}S_L}{A_L} - \frac{\tau_{iL}S_i}{A_L} - \rho_L g \sin\beta \qquad [9]$$

and

$$-\left(\frac{dP}{dx}\right)_{TPG} - \frac{\alpha\rho_G}{2}\frac{d(V_G^2)}{dx} = \frac{\tau_{WG}S_G}{A_G} + \frac{\tau_{iG}S_i}{A_G} - \rho_G g \sin\beta, \qquad [10]$$

where β is the tube inclination from the horizontal.

These equations can be put into dimensionless form; however no new information is found in either [9] or [10] or in the dimensionless equation. For example if both the ILG and tube inclination are downward, the dimensionless equation is simply

$$\chi^2 F(R_L, n, m) - F\left(R_L, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right) - Z - 4Y = 0. \qquad [11]$$

This is [8a] with the Z parameter which accounts for ILG and with the Y parameter introduced by Taitel & Dukler (1976a) to account for tube inclination.

Of considerable interest are similarities between open-channel flow and stratified gas–liquid flow. Equation [9] combined with the continuity equation, can be written in a form similar to that used in open-channel flow to produce [12]. The result defines more clearly the hydrodynamic characteristics of two-phase stratified flow and the ILG:

$$\frac{\mathrm{d}h_\mathrm{L}}{\mathrm{d}x}=\frac{S_0-S}{1-\mathrm{Fr}^2}, \qquad [12]$$

where

$$S_0=\sin\beta,$$

$$S=\frac{\tau_{\mathrm{WL}}S_\mathrm{L}}{gA_\mathrm{L}\rho_\mathrm{L}}-\frac{\tau_{\mathrm{iL}}S_\mathrm{i}}{gA_\mathrm{L}\rho_\mathrm{L}}+\frac{1}{g\rho_\mathrm{L}}\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPL}}$$

and

$$\mathrm{Fr}^2=\frac{\alpha V_\mathrm{L}^2\psi}{gA_\mathrm{L}}, \quad \psi=\frac{\mathrm{d}A_\mathrm{L}}{\mathrm{d}h_\mathrm{L}}.$$

Equation [12] is compact and provides the following information:

(1) ILG will always exist unless $S_0-S=0$, or $S_0=0$ and $S=0$;
(2) if ILG exists, stratified liquid–gas flow can be subcritical, critical or supercritical according to whether the Froude number (Fr) is less than, equal to or greater than unity.

To date, stratified flow has not been investigated from this viewpoint because it has been generally assumed that stratified flow is uniform. Supercritical stratified flow would be indicated by an upward slope of the interfacial gradient and $\mathrm{Fr}>1$.

In open-channel flow there is no axial pressure gradient and if the duct is horizontal and flow is very subcritical, [12] reduces simply to

$$\frac{\rho_\mathrm{L}g\dfrac{\mathrm{d}h_\mathrm{L}}{\mathrm{d}x}}{S}=1. \qquad [13]$$

Thus the driving force due to ILG just balances the frictional resistance for flow through the channel. The counterpart of [13] for stratified flow can be expressed in terms of the measured pressure gradients:

$$\frac{\left(\dfrac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPLM}}}{\left(\dfrac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPGM}}}=\Sigma^2. \qquad [14]$$

The dimensionless parameter Σ^2 is a numerical indication of ILG and is a direct measure of the effective magnitude of ILG.

Another comparison between open-channel and stratified flow involves the determination of the relative magnitude of h_{EL}, the liquid height at the free overflow location. Over a wide range of circular pipe diameters, Smith (1963) reported data giving values of h_{EL} for flow through horizontal tubes:

$$\frac{Q}{(gD^5)^{1/2}}=1.55\left(\frac{h_{\mathrm{EL}}}{D}\right)^{1.88}, \qquad [15]$$

where Q is the volumetric flow rate and D is the tube diameter. In the current analysis, values of h_{EL} in stratified flow are compared with those predicted by [15]. Observe that [15] indicates that h_{EL} is independent of the channel length.

In subsequent figures found in the results and discussion section, holdup and ϕ_L^2 were calculated using uniform flow equations [1] and [2] and the assumption $f_{\mathrm{iG}}=f_{\mathrm{iL}}$. Comparison between open-channel and stratified flow with ILG employed [12] and [15].

Recently, flow-pattern maps have been presented for horizontal gas–liquid flow through circular tubes by Taitel & Dukler (1976b) and Weisman *et al.* (1979). Although both the maps are for uniform flow and do not consider ILG, the Taitel–Dukler map was selected as a reference simply for convenience. The current analysis explores only the transitions from smooth stratified (SS) to wavy stratified (WS) flow and from wavy stratified to intermittent (I) flow.

In the current analysis of the effect of ILG, it should be remembered that in general, average values of holdup were used and hence averaged ILG effects were determined. The actual ILG profile requires numerical analysis to obtain local values similar to the step-by-step analysis required to determine local liquid depths in open-channel flow (Henderson 1966).

RESULTS AND DISCUSSION

Gazley (1948, 1949) and Holden (1948) reported the only ILG profiles found in the literature. Gazley's and some of Holden's holdup data are compared in figure 2 with the uniform flow predicted values. For each liquid flow rate, experimental holdup is substantially lower than the predicted value at the higher values of χ (lower gas flow rates); moreover decreasing χ has virtually no effect on holdup until holdup finally merges to the predicted χ_{lt} curve. One explanation for this behavior is that initially, increased gas flow rate acts primarily to depress the ILG rather than to reduce the average liquid level (or holdup). A comparison of the Holden and Gazley holdup data suggests that where ILG exists there might be a diameter effect on holdup, whereas if uniform flow exists no effect of diameter is predicted. Although all the holdup data converges in the limit to predicted uniform flow values, they converge to the χ_{lt} curve whereas convergence to the χ_{tt} curve would have been expected because the data showed that at low χ values both phases were turbulent. No conclusive effect of ILG on flow pattern can be observed when the Gazley stratified data are placed on the Taitel–Dukler horizontal uniform stratified flow map in figure 3 (K and F are flow-pattern parameters of Taitel & Dukler). The fact that near the SS–WS boundary observed SS data fall in the WS region might be attributed to the uncertainty in the predicted WS boundary.

Analysis of the Jensen (1972) data uncovered several important features related to high-viscosity liquid–gas stratified flow. Although not stated explicitly in Jensen's thesis, the current analysis concludes that the liquid flow rates were experimentally selected to reduce or eliminate ILG. To obtain uniform flow conditions it was necessary to use extremely low liquid velocities and high gas velocities because the Jensen test L/D (tube length-to-diameter ratio) was relatively small and the liquid viscosity was as high as 310 cP. (Significant ILG is expected for high liquid viscosity flow through relatively short tubes.) Accurate elimination of ILG was possible because the test section was transparent and the liquid level was measured electronically and mechanically. As a result, a large majority of

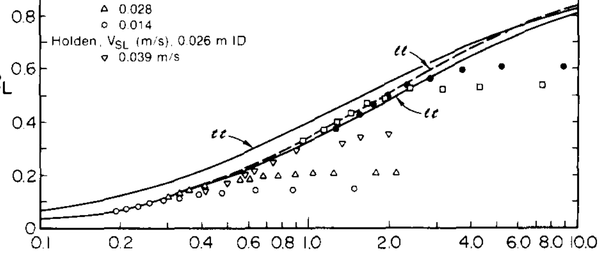


Figure 2. Comparison of predicted uniform stratified liquid holdup with the experimental data of Gazley (1948) and Holden (1948).

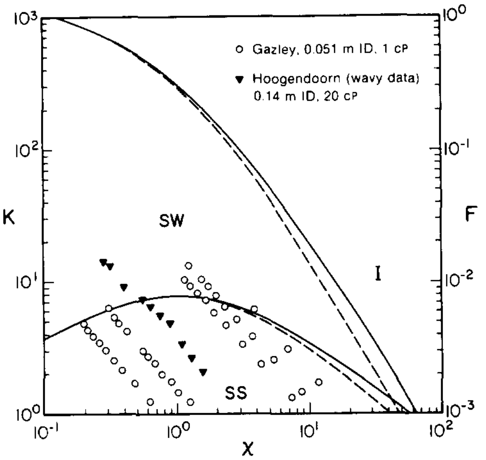


Figure 3. Comparison of experimental data (Gazley 1948; Hoogendoorn 1959) with the Taitel–Dukler (1976b) boundary between smooth and wavy stratified flow patterns.

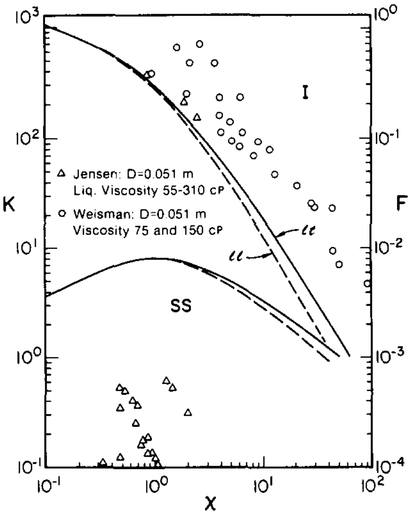


Figure 4. Predicted flow patterns for observed smooth stratified flow (Jensen 1972; Weisman 1979).

their high-viscosity data were well within the smooth stratified flow region, as seen in figure 4. (Other data not shown fall below $K = 0.1$ in figure 4.) Moreover their holdup R_L data and two-phase pressure drop ratio ϕ_L^2, seen in figures 5 and 6 respectively, compare very favorably with the predicted values of [3] for uniform horizontal stratified flow having no ILG. Typical calculated parameters for the 250 cP liquid viscosity data are shown in table 3. On the assumption of $\tau_{iL} = \tau_{iG}$, a significant difference between $(\Delta P/\Delta L)_{TPL}$ and $(\Delta P/\Delta L)_{TPG}$ is a quantitative indication of ILG. (For the Jensen and other data analyzed it was generally found that $\tau_{iL} \simeq \tau_{iG}$ for uniform smooth stratified flow.) Gas velocities as high as 10 m/s were required to suppress ILG at liquid-phase superficial velocities as low as 0.00037 m/s ($R_L = 0.113$). This condition resulted in a gas-to-liquid velocity ratio as high as 2.7×10^4. Jensen (1972) observed that if the liquid viscosity exceeded approx. 240 cP, the wave and slug transitions appeared to occur simultaneously. Any tendency for wave formation erupted into slugs. Thus at high liquid viscosities a stable wavy flow pattern did not exist. Use of large-diameter pipes or low-viscosity liquids would tend to reduce this abrupt slugging. Deshpande & Bishop (1983) observed similar behavior in non-Newtonian liquid–gas stratified flow.

The effect of ILG on flow-pattern determination is illustrated vividly by comparing the flow-pattern observations of Weisman *et al.* (1979) for 75 and 150 cP liquid against the observed flow-pattern data of Jensen (1972) for 55–310 cP liquid viscosity. The comparison is given in figure 4. Although Weisman observed smooth stratified flow, use of the Taitel–Dukler flow transition parameters indicate that the data near the boundary are in

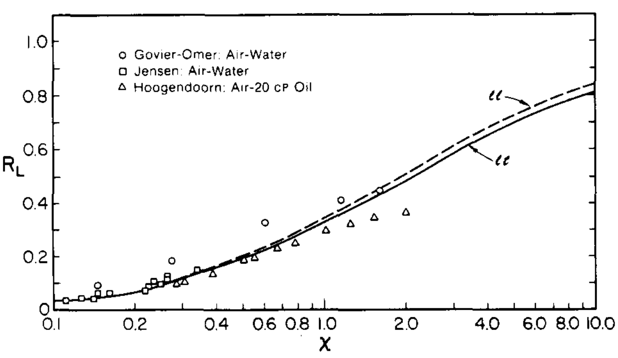


Figure 5. Liquid holdup R_L vs the Lockhart–Martinelli χ parameter in stratified flow (Jensen 1972).

Table 3. Smooth stratified horizontal flow

Data point	V_{SG} (m/s)	Re_L	$Re_G \times 10^{-4}$	x	ϕ_L^2	R_L[a]
1	2.6	0.97	1.0	1.4	—	0.41
2	3.2	0.91	1.3	1.3	—	0.37
3	3.7	0.93	1.4	1.1	—	0.35
4	4.5	1.0	1.7	0.94	3.1	0.32
5	5.8	1.1	2.2	0.78	2.9	0.28

Data point	τ_{WL} (N/m²)	τ_{iG} (N/m²)	τ_{WG} (N/m²)	ILG	$\left(\frac{\Delta P}{\Delta x}\right)_{TPL}$ (N/m³)	$\left(\frac{\Delta P}{\Delta x}\right)_{TPG}^{a}$ (N/m³)
1	0.18	0.066	0.091	Yes	11.69	9.42
2	0.27	0.091	0.12	Yes	18.79	12.01
3	0.29	0.12	0.14	Yes	19.28	14.35
4	0.30	0.17	0.18	No	17.20	18.77
5	0.41	0.25	0.25	No	23.56	24.68

[a]Data from Jensen (1972).
Liquid: glycerine–water
Tube diameter: 0.051 m
$V_{SL} = 0.0018$ m/s
Liquid viscosity: 250 cP
TS length: 7.3 m
Gas: air

the intermittent flow region. This result was interpreted by Weisman *et al.* (1979) to imply that the Taitel–Dukler WS–I boundary is not valid for high-viscosity liquid–gas stratified flow. However, as seen in figure 4, the 55–310 cP viscosity data of Jensen conform to the predicted SS flow pattern. Based on the current analysis it is suggested that significant ILG existed in Weisman's data. The effect of downward ILG is similar to downward tube inclination as seen from [8a]; both act to expand the stratified flow region. Weisman *et al.* presented a different expression which contains the ratio of the superficial gas to superficial liquid velocities to predict transition from wavy stratified to intermittent flow. The criterion predicts WS–I transition at considerably higher liquid velocities than predicted by Taitel & Dukler for high-viscosity liquids. In formulating their criterion, Weisman *et al.* used a gas-to-liquid velocity ratio as high as 100, whereas Jensen used velocity ratios higher than 10^4 to partially suppress ILG. The conclusion is that for a high-viscosity liquid the Taitel–Dukler transition criteria are not valid for non-uniform flow or stratified flow with ILG. Conversely, the Weisman WS–I criterion is not valid for

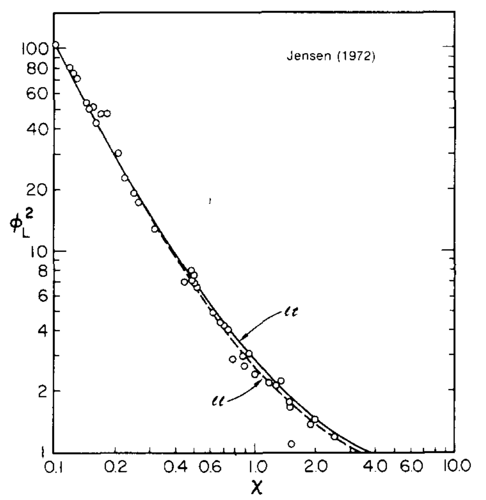


Figure 6. Liquid two-phase pressure drop parameter ϕ_L^2 vs the χ parameter in uniform stratified flow (Jensen 1972).

uniform flow without ILG. Because of the very low liquid velocities and high gas velocities employed to satisfy uniform stratified flow requirements for high-viscosity liquids, Jensen's results illustrate the large effect of liquid viscosity on stratified to intermittent flow-pattern transition; nevertheless, excellent agreement with uniform stratified flow predictions can be achieved. When high-viscosity liquid stratified flow is obtained in relatively short tubes, high ILG will generally result unless deliberate action is taken to suppress it. Such action can be to drastically increase the gas velocity.

Hoogendoorn (1959) reported a limited number of wavy stratified flow data for a 20 cP oil–air mixture flowing through a 0.14 m dia pipe. The liquid velocity was 0.072 m/s, and the air velocity range was 3.2–22 m/s. Other parameters are listed in tables 1 and 2. No smooth stratified flow data were reported for the liquid viscosity, liquid flow velocity and diameter tube cited. It is not clear why smooth stratified flow data could not have been obtained for the liquid velocity used simply by reducing the gas velocity. It is hypothesized that ILG existed at a gas velocity equal to 3.2 m/s and became very significant below that velocity. To test the hypothesis one extrapolated data point was secured at a lower gas velocity. Because the flow was wavy stratified the assumption of $\tau_{iL} = \tau_{iG}$ is not valid, however, the assumption $(\Delta P/\Delta x)_{TPL} = (\Delta P/\Delta x)_{TPG}$ should remain valid even in ripple–wavy stratified flow if the flow is uniform, the irreversible loss of energy at the interface is small and no ILG exists. Calculations for the first four data points and the extrapolated data point are given in table 4. It is seen that indeed there is a crossover in the range of 3.2–2.2 m/s from $\tau_{iL} < \tau_{iG}$ to $\tau_{iL} > \tau_{iG}$; however at both velocities $\tau_{iG} > \tau_{WG}$. This result indicates a change from uniform wavy flow to wavy flow with ILG. Thus wavy flow can coexist with ILG. Additional evidence of these conclusions is seen in figure 7 where the characteristic behavior of holdup under stratified flow conditions is observed. The holdup is lower than predicted by uniform flow analysis and at the high values of χ, R_L tends to be somewhat insensitive to changes in χ. Figure 8 also shows that in the region under study the calculated value of ϕ_L^2 crosses the value predicted by uniform flow analysis. Although Hoogendoorn reported all of this data to be wavy stratified, use of the Taitel–Dukler map predicts most of the data (7 out of 10 points) in the smooth stratified region (figure 3).

There are several reasons why the Taitel–Dukler SS–WS transition criterion might

Table 4. Wavy stratified horizontal flow with ILG

Data point	V_{SG} (m/s)	Re_L	$Re_G \times 10^{-4}$	χ	ϕ_L^2	R_L[a]
1	7.0	1190	7.1	0.77	6.2	0.25
2	5.0	1100	5.2	1.0	3.8	0.30
3	4.1	1090	4.2	1.3	2.7	0.31
4	3.2	1030	3.4	1.5	1.9	0.35
5[b]	2.2	1010	2.4	2.1	1.0	0.37

Data point	τ_{iL} (N/m^2)	τ_{WL} (N/m^2)	τ_{iG} (N/m^2)	τ_{WG} (N/m^2)	ILG	$\left(\frac{\Delta P}{\Delta x}\right)_{TPG}^{a}$ (N/m^3)
1	0.17	0.50	0.82	0.25	No	15.25
2	0.18	0.38	0.43	0.16	No	9.13
3	0.24	0.36	0.30	0.11	Uncertain	6.50
4	0.22	0.30	0.19	0.085	Probably	4.63
5[b]	0.28	0.28	0.095	0.047	Yes	2.50

[a]Data from Hoogendoorn (1959).
[b]Extrapolated.
Liquid: oil
Tube diameter = 0.14 m
V_{SL} = 0.072 m/s[a]
Liquid viscosity: 20 cP
TS length = 8 m
Gas: air

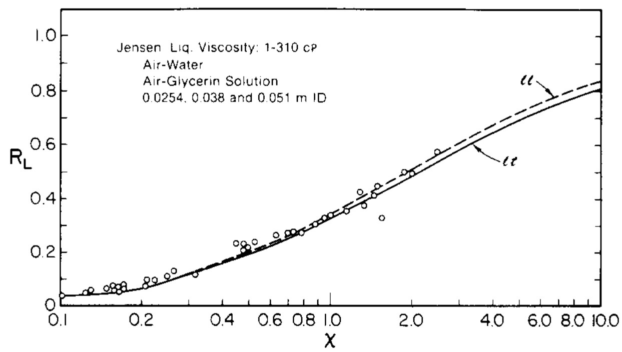


Figure 7. Liquid holdup as a function of the χ parameter in stratified flow (Govier & Omer 1962; Jensen 1972; Hoogendoorn 1959).

overpredict the range of the smooth stratified flow region:

(1) ILG delays the transition to wavy stratified flow; therefore it would have been expected that more of the Gazley data would have been predicted to be in the wavy stratified flow region.
(2) The smooth stratified flow data of Jensen fall far below the SS–WS line. Because of the conditions used to obtain the smooth stratified flow data (discussed previously), it would have been expected that the Jensen data would fall nearer to the WS–SS boundary.
(3) The Hoogendoorn data were specified to be wavy stratified yet the majority of them fall in the predicted smooth stratified flow region.

It is possible that the WS–SS transition boundary (or the value of the flow pattern parameter K) is high by a factor of 10. Taitel & Dukler (1976b) realized the uncertainty of the WS–SS boundary because of the difficulty in assigning a consistent value of S, the sheltering coefficient in the analysis (Jeffreys 1925, 1926).

Calculated values of shear stresses derived from the assumption of uniform flow are indicative of various stratified flow types, as follows:

$\tau_{iL} \simeq \tau_{iG} \simeq \tau_{WG}$ uniform stratified flow, no ILG
$\tau_{iL} > \tau_{iG} \simeq \tau_{WG}$ non-uniform stratified flow with ILG
$\tau_{iL} < \tau_{iG} > \tau_{WG}$ wavy stratified flow with or without ILG.

The consistency of these results provides justification for defining the respective equivalent diameters of the gas and the liquid phase as

$$D_G^e = \frac{4A_G}{S_G + S_i} \quad \text{and} \quad D_L^e = \frac{4A_L}{S_L}, \qquad [16]$$

where D^e is the equivalent diameter. Other definitions have been used (Govier & Aziz 1972).

When ILG exists, the measured two-phase pressure gradients in the liquid and gas phases are not equal; therefore, use of the Lockhart–Martinelli parameters ϕ_L^2 and ϕ_G^2 is not valid because they have no common reference. Nevertheless one still finds their use in the literature of stratified flow in spite of the fact that the results can be misleading. For example, if ϕ_L^2 is calculated using the measured two-phase gas pressure gradient, a ϕ_L^2 less than unity might result, whereas the use of the measured two-phase liquid pressure gradient will always give a higher calculated value for ϕ_L^2. Agrawal's stratified flow information is unique because these are the only data found which indicate ϕ_L^2 (based on the measured centerline pressure gradient) is less than unity. This drag-reduction behavior had been predicted by Heywood & Charles (1979) using the assumption of uniform stratified flow. The analysis of this paper predicts that ILG occurred in some of the Agrawal data. This result, together with higher values of ϕ_L^2 based on the calculated

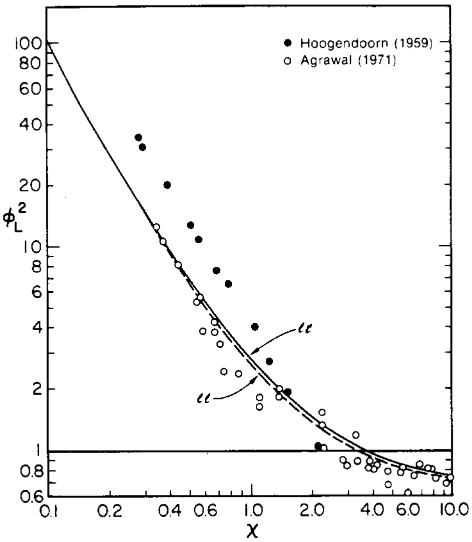


Figure 8. Liquid two-phase pressure drop parameter ϕ_L^2 vs the χ parameter in stratified flow.

two-phase liquid pressure gradient, are found in table 5. The values of ϕ_L^2 based on measured centerline pressure gradients are also shown in figure 8. Because of the calculational uncertainties and the possible errors in pressure drop measurements (several repeat pressure drop measurements differed by as much as 50% from the original data), the question of two-phase drag reduction in Newtonian liquid–gas stratified flow is unresolved. These results reinforce the need to measure the pressure gradient in each phase during stratified flow experiments and the difficulty in obtaining consistently accurately measured pressure gradients using centerline pressure taps if ILG is present. However, the majority of the Agrawal holdup data, seen in figure 9, fall on the uniform stratified flow curve. Thus liquid holdup may not be as sensitive a measure of ILG as pressure drop. A prediction of only minor ILG in the Agrawal data is not surprising because although the liquid viscosity was 5.0 cP, the tube L/D was 1170. This L/D was the highest value found in any of the tests. All of the Agrawal data fall in the smooth stratified region although he reported both smooth stratified and wavy stratified flow patterns. This result supports the earlier contention that the Taitel–Dukler flow-pattern map probably overpredicts the smooth stratified flow region (figure 10).

Table 5. Comparison of the two-phase pressure gradient parameter ϕ_L^2

System: oil–air; Tube diameter: 0.0258 m; $V_{SL} = 0.0216$ m/s[a]
Liquid viscosity = 5.0 cP; TS length = 17.1 m; $(\Delta P/\Delta x)_{SL} = 5.28$ N/m^3

Data point	V_{SG}[a] (m/s)	Re_L	Re_G	χ	$(\phi_L^2)_{TPL}$	$(\phi_L^2)_{TPG}$
1	0.111	136	307	7.42	0.87	0.5
2	0.197	142	512	5.58	0.895	0.78
3	0.357	147	882	4.14	0.944	0.86
4	0.777	162	1745	2.81	1.045	0.88

Data point	τ_{WL} (N/m^2)	τ_{iG} (N/m^2)	τ_{WG} (N/m^2)	ILG	$\left(\frac{\Delta P}{\Delta x}\right)_{TPL}$ (N/m^3)	$\left(\frac{\Delta P}{\Delta x}\right)^a_{TPG}$ (N/m^3)
1	0.036	0.001	0.009	Large	4.62	2.64
2	0.038	0.008	0.012	Small	4.73	4.12
3	0.041	0.008	0.016	Small	4.99	4.57
4	0.051	0.008	0.021	Small	5.52	4.66

[a]Data from Agrawal (1971).

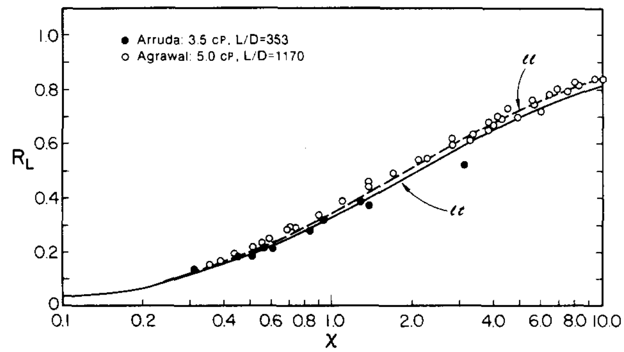


Figure 9. Comparison of liquid holdup data with uniform stratified flow predictions (Arruda 1970; Agrawal 1971).

Arruda (1970) obtained his data in order to determine criteria for the SS–WS flow transition. Figure 9 illustrates that over the entire range of χ, where the two sets of data can be compared, the Arruda holdup data are lower than both the Agrawal data and the uniform stratified flow prediction. Moreover, the characteristic insensitivity induced by ILG is seen. The tube L/D was only 353 compared to 1170 in the Agrawal tests; the tube diameter was the same, 0.026 m, in both sets of tests. It would have been expected that in figure 10 the flow-pattern data would straddle the SS–WS boundary; however, all the data are in the smooth stratified region, another indication that the predicted boundary overpredicts the smooth stratified region.

No ILG was detected in the current analysis of the five stratified data points reported by Govier & Omer (1962). All holdup data fall above the prediction, as shown in figure 7. The fact that the L/D was approx. 500 and water was used mitigates against ILG. Govier & Omer indicated that four of their data points were smooth stratified and the fifth was wavy stratified. The predicted values are found in figure 10.

Simpson *et al.* (1976, 1981) reported stratified flow data for 0.127 and 0.216 m dia tubes. These data are very important for several reasons:

(1) The tube diameters are the largest reported.
(2) The liquid Reynolds numbers are highly turbulent; the air Reynolds numbers ranged from laminar to highly turbulent. In many tests the liquid velocity was higher than the gas velocity.

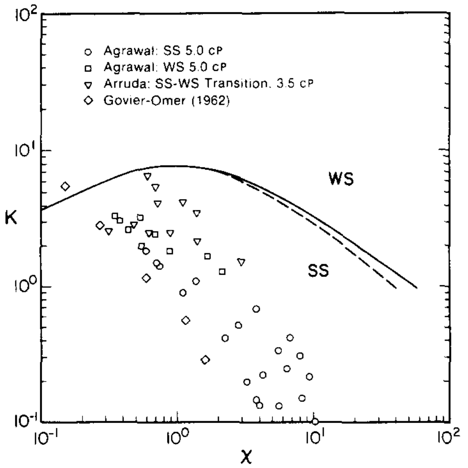


Figure 10. Observed flow patterns (Agrawal 1971; Arruda 1970; Govier & Omer 1962) vs predicted flow patterns.

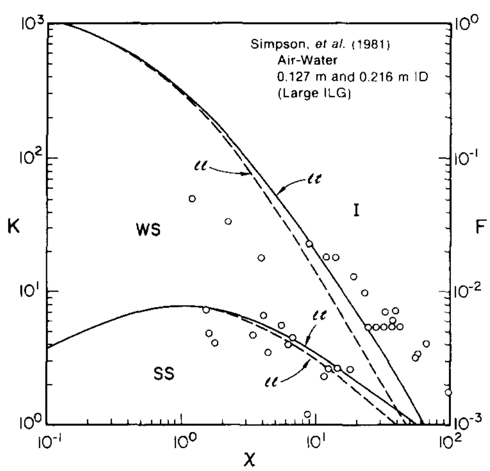


Figure 11. Comparison of observed stratified flow-pattern data (Simpson *et al.* 1981) with the predicted flow patterns.

(3) Measurements of the axial two-phase pressure gradient were made in both the liquid and gas phases and along the tube centerline.
(4) This is the only set of experiments found which used an MT (as defined in the introduction).

The measured pressure gradients in the liquid phase were much higher than the gas-phase pressure gradient measurements. Pressure gradient measurements obtained from the centerline taps were often in error. The maximum value obtained for Σ^2 was 207, an indication of extremely high ILG. Simpson *et al.* (1976) recognized that high ILG existed. Flow patterns observed to be stratified are predicted to be stratified and intermittent, as seen in figure 11. This characteristic stratified flow-pattern expansion by ILG, which has been referred to before, appears to be pronounced in these experiments. Again, over a wide range of χ it is seen that there is only a very small change in liquid holdup, another indication of ILG (figure 12). The holdup values were measured at one location. The effect of ILG is best expressed by Σ^2 rather than the absolute value of ILG, where $\Sigma^2 = (dP/dx)_{TPLM}/(dP/dx)_{TPGM}$; $(dP/dx)_{TPLM}$ is the measured value of the axial pressure gradient in the liquid phase and includes $\rho_L g dh_L/dx$.

It would be desirable to have a correlation between Σ^2 and other parameters which enter stratified flow analysis. The difficulty is that only Simpson *et al.* measured the pressure gradient in each phase. More recently, Deshpande & Bishop (1983) measured Σ^2 in non-Newtonian liquid–gas stratified flow. Preliminary results indicate that if $\chi^2 < 1$, $\Sigma^2 = 1$ (also see table 3). Examination of the existing Newtonian liquid–gas stratified flow data suggests the criterion is valid; however, more data and analysis are required before a conclusion can be reached.

Similarities between stratified flow and open-channel flow through circular duct were investigated using [12] and [15] to predict the occurrence of critical flow and the height of liquid at the overflow location. Interfacial profiles are shown for two liquid flow rates in figure 13 from Bergelin & Gazley (1949). In the top figure there is a change from subcritical to supercritical flow, as indicated by the change in the direction of the slope from negative to positive. In the bottom figure flow is subcritical for all gas flow rates used. In open-channel flow such a change is predicted by a Froude number equal to unity in [12]. Our calculated results using predicted h_{EL} values are given in table 6. The agreement with Fr = 1 for the transition from subcritical to supercritical flow is good. The principal uncertainty is the correct value to be used for the laminar flow kinetic energy correction factor for open-channel flow. For the data in the bottom figure the kinetic energy correction factor is essentially unity because the liquid flow is turbulent. Liquid depths at the overflow locations were also calculated using [15] and compared with the extrapolated experimental data at location 20. These values of the overflow depth differ by a factor of 2. It should be realized however that location 20 is not the actual experimental overflow

Table 6. Subcritical and supercritical stratified flow

Water flow rate (kg/h)	Air velocity V_G (m/s)	Water velocity V_L (m/s)	Water Reynolds No., Re_L	Air Reynolds No., Re_G	Liquid Holdup, R_L	Liquid depth at end (m), h_{EL} (predicted)	Liquid depth at location 20[a] (m), h_{EL} (experimental)	Froude No., Fr
110.3	0	0.102	2660	0	0.140	0.00454	0.00983	0.83
110.3	1.49	0.114	2728	4719	0.125	0.00454	0.00927	0.49
110.3	1.84	0.121	2766	5895	0.118	0.00454	0.00907	1.05
110.3	2.19	0.130	2807	7079	0.110	0.00454	0.00869	1.13
110.3	2.53	0.139	2850	8236	0.103	0.00454	0.00838	1.21
830.8	0	0.214	11,438	0	0.500	0.0133	0.0249	0.71
830.8	2.60	0.228	12,730	5562	0.470	0.0133	0.0244	0.76
830.8	3.15	0.233	12,025	6899	0.460	0.0133	0.0239	0.77
830.8	4.37	0.274	12,868	10,627	0.390	0.0133	0.0212	0.91
830.8	5.36	0.306	12,780	14,195	0.350	0.0133	0.0195	1.02

[a]"Location 20" is 20 ft from the end of the entrance section.

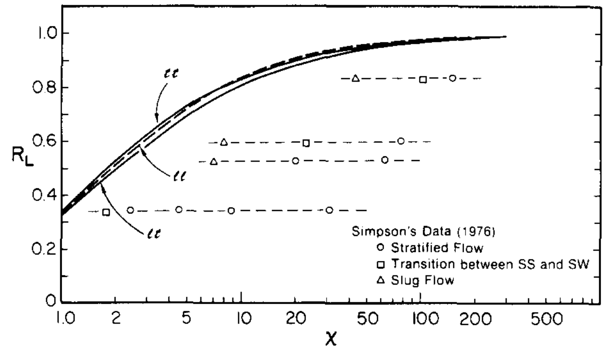


Figure 12. Comparison of Simpson *et al.*'s (1976) liquid holdup data with the predictions of uniform flow.

location. These limited results suggest that in regard to the criteria for the transition from subcritical to supercritical flow, stratified gas–liquid flow can be analyzed as open-channel flow.

In summary, it has been shown that ILG existed or tended to exist over a wide range of stratified two-phase liquid–gas data reported to date. The presence of ILG is caused primarily by the liquid phase attempting to flow independent of the gas phase. Such a condition would occur in horizontal open-channel flow where a decrease in the liquid level would be required to balance the steady-state frictional resistance. There are two possible effects of ILG; the stratified flow region might be expanded and the use of the Lockhart–Martinelli two-phase pressure drop parameters ϕ_L^2 and ϕ_G^2 is invalid. Stratified flow having ILG is a type of non-uniform flow which is not restricted to situations where high-viscosity liquids, combining tees, short test sections or small-diameter tubes are used. Nevertheless, it does appear that the magnitude of ILG is accentuated by high values of χ^2, low gas-to-liquid flow ratios, low total length-to-diameter test section ratios and the use of CTs (in contrast to the use of MTs). Additional work is required to determine the definitive effect upstream (CTs) and downstream conditions have on the magnitude of ILG. Moreover, analysis should also be performed to establish specific ILG criteria. For example it is suggested here that if $\chi^2 < 1$, ILG is minimal and $\Sigma^2 = 1$; whether this preliminary criterion is generally valid and is independent of the geometry should be investigated. The work should be extended to non-Newtonian liquid–gas stratified flow. Finally, in any future stratified flow experiments, the axial pressure gradient should be measured in each phase. Centerline pressure taps should not be used if ILG is suspected. Ideally, some provisions should be included to measure the height of the liquid level.

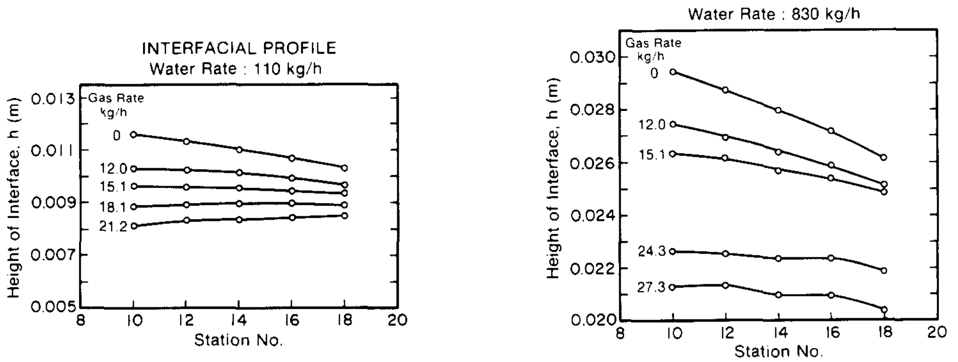


Figure 13. ILGs in subcritical and supercritical flow [experimental liquid level profiles reported by Gazley (1948) and Bergelin & Gazley (1949)].

CONCLUSIONS

1. Stratified flow liquid holdup and two-phase pressure drop can be predicted using the dimensionless form of the one-dimensional energy equations if the stratified flow is uniform without ILG.
2. Analysis indicated that many of the previously reported stratified flow data exhibited non-uniform (ILG) behavior in varying degrees.
3. If high-viscosity liquids are used in stratified flow tests, extremely high gas-to-liquid velocity ratios are required to reduce the magnitude of ILG or to eliminate it completely.
4. ILG can exist even if initially, the two phases are combined using a device which thoroughly mixes the phases.
5. ILG probably reduces the liquid holdup, expands the stratified flow regime and delays the transition to wavy stratified and/or intermittent flow.
6. In stratified flow experiments, the pressure gradient should be measured in each phase because the parameter $(\Delta P/\Delta x)_{\mathrm{TPLM}}/(\Delta P/\Delta x)_{\mathrm{TPGM}} = \Sigma^2$ is a quantitative measure of ILG, even if ILG is not observed visually.
7. Use of the Lockhart–Martinelli parameters ϕ_L^2 and ϕ_G^2 is not valid if ILG is present because there is no common reference two-phase axial pressure gradient. Therefore, any attempts to use a ϕ_L^2 or a ϕ_G^2 vs χ relationship is meaningless.
8. If measured pressure gradient data for both the phases are not available and the uniform flow energy equations are assumed to be valid for any arbitrary set of data, the following results are indicative of the actual stratified flow regimes:

(a) $\tau_{iL} \simeq \tau_{iG} \simeq \tau_{WG}$ uniform stratified flow, no ILG
(b) $\tau_{iL} > \tau_{iG} \simeq \tau_{WG}$ nonuniform stratified flow with ILG
(c) $\tau_{iL} < \tau_{iG} > \tau_{WG}$ wavy stratified flow with or without ILG.

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NOMENCLATURE

A = Area
D = Diameter
D^e = Equivalent diameter
F = Function or flow-pattern parameter of Taitel & Dukler (1976b)
f = Friction factor
Fr = Froude number
g = Acceleration due to gravity
h = Height
I = Parameter defined in [6]
K = Flow-pattern parameter of Taitel & Dukler (1976b)
L = Length
m = Exponent in friction factor relationship for the gas phase
n = Exponent in friction factor relationship for the liquid phase
P = Pressure
R = Holdup
S = Perimeter or parameter introduced in [12] or sheltering coefficient
S_0 = Parameter introduced in [12]
V = Velocity
x = Length
Y = Parameter introduced by Taitel & Dukler (1976a), used in [11]
Z = Parameter defined in [8b]

Subscripts

G = Gas
i = Interfacial
iG = Interfacial gas
iL = Interfacial liquid
L = Liquid
lt = Laminar–turbulent
SG = Superficial gas
SL = Superficial liquid
TPG = Two-phase gas
TPL = Two-phase liquid
TPGM = Two-phase gas measured
TPLM = Two-phase liquid measured
tt = Turbulent–turbulent
WG = Wall gas
WL = Wall liquid

Greek symbols

χ = Lockhart–Martinelli parameter
τ = Shear stress
α = Kinetic energy correction factor
ρ = Density
ϕ_L^2 = Two-phase pressure drop parameter
Σ^2 = Parameter defined in [14]
β = Tube inclination from the horizontal
Ω = Inclination of gas–liquid interface

Abbreviations

CC = Centerline
CT = Combining tee
G–W = Glycerine water solution
I = Intermittent
ILG = Interfacial level gradient
MT = Mixing tee
SCT = Special combining tee
SS = Smooth stratified
S-ILG = Stratified flow with interfacial level gradient
TS = Test section
W = Water
WS = Wavy stratified

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